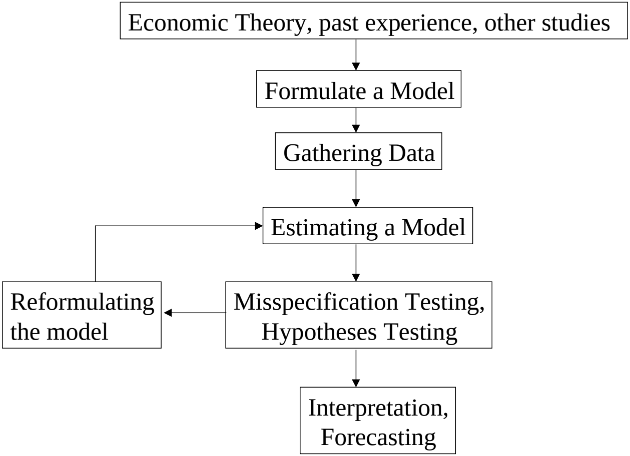
*The Nature of Econometrics and Economic Data*[Wooldridge (2013) Chapter 1 and Chapter 2 (sections 2.1 and 2.2)]

- Major uses of Econometrics
- Basic Ingredients of an empirical project
- Formulate a model (example)
- The Question of Causality
- Misspecification Testing
- Types of Data
- The Simple Regression Model
    - Introduction
    - Ordinary Least Squares (OLS)
    - Deriving OLS Estimates
    - Alternative approach to derivation
    - Some definitions

1. Describing Economic Reality.
2. Testing hypotheses about Economic Theory.
3. Forecast future economic activity.

**Flow chart for the Steps of an Empirical Study**




**Remark:** This module is not about Economic Theory and gathering data.

Economic Theory suggests interesting relations between variables.

**Example:** Returns to education

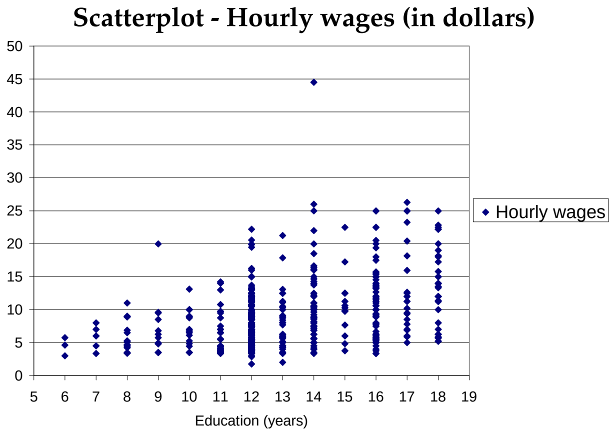
- A model of human capital investment predicts that getting more education should lead to higher wages:

$$w = f(E)$$

  where $w$ are the wages of a person and $E$ are years of education of the person,

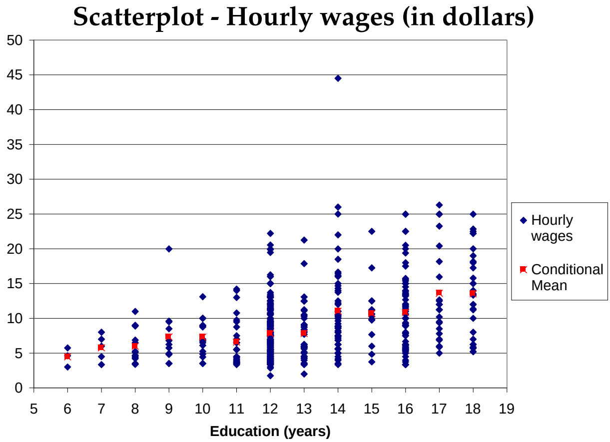
$$\frac{\partial w}{\partial E} > 0.$$

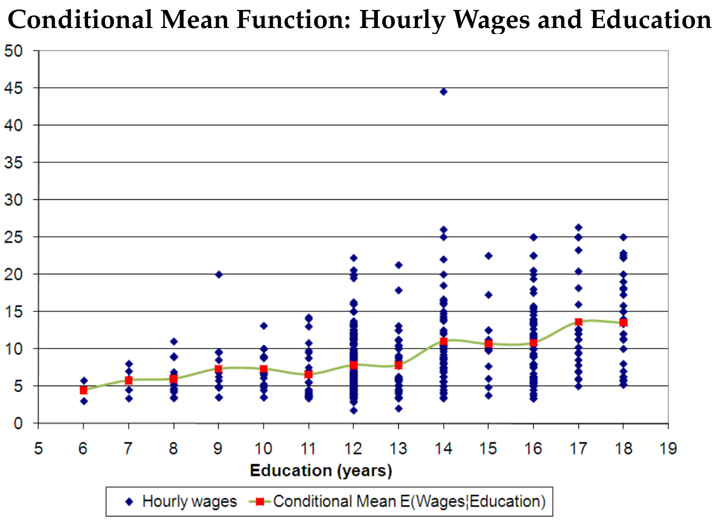
- However, let us look at a data set: US national survey of people in the labour force that already completed their education, 528 people.




- People with the same years of education earn different hourly wages.
- There is a distribution for the hourly wages conditional on the years of education.

- How can we study if the evidence of the data supports Economic Theory?
- A possibility is to look at means of wages conditional on the years of Education.






We can see that the mean of wages varies with the years of Education.

- Hence, the object that we are interested in studying is the mean of wages given the years of Education: $E\left[Wages|Education\right]$.
- To simplify computations and the interpretation of results usually we assume a model for $E\left[Wages|Education\right]$.
- A possible model for $E\left[Wages|Education\right]$ is

$$E\left[Wages|Education\right] = \beta_0 + \beta_1 Education.$$

- Notice that for any value $a$

$$\begin{aligned} & E\left[Wages|Education = a+1\right] - E\left[Wages|Education = a\right] \\ = \ & \beta_0 + \beta_1 (a+1) - \beta_0 - \beta_1 a \\ = \ & \beta_0 + \beta_1 a + \beta_1 - \beta_0 - \beta_1 a \\ = \ & \beta_1. \end{aligned}$$

Hence, $\beta_1$ is the change of the expected value of *Wages* for one additional year of Education.

- Equivalently, the model can be written in the more familiar way

$$Wages = \beta_0 + \beta_1 Education + u,$$

where $E\left[u|Education\right] = 0$.

- To see this notice that

$$Wages = E\left[Wages|Education\right] + Wages - E\left[Wages|Education\right]$$

- Let $u = Wages - E\left[Wages|Education\right]$, therefore

$$\begin{aligned} Wages &= E\left[Wages|Education\right] + u \\ &= \beta_0 + \beta_1 Education + u, \end{aligned}$$

Now notice that by construction

$$\begin{gathered} E(u|Education) = E\{Wages - E\left[Wages|Education\right]|Education\} \\ = E\left[Wages|Education\right] - E\{E\left[Wages|Education\right]|Education\} \\ = E\left[Wages|Education\right] - E\left[Wages|Education\right] \\ = 0. \end{gathered}$$

$$Wages = \beta_0 + \beta_1 Education + u,$$

where $E\left[u|Education\right] = 0$.

- $u$ is denoted the *error term*.
- This model is known as *The Simple Regression Model*.
- It is linear in the parameters $\beta_0$ and $\beta_1$.

The estimate of $\beta_1$, is the return to education, but can it be considered causal?

- We would like to prove that the effect is causal.
- However it is impossible to prove causality. If $\beta_1 \neq 0$ and we have a sound theoretical economic argument, this might indicate that there is a causal relation. However this is far from being a proof.

**Major challenges:**

- Inference procedures depend of the characteristics of the distribution of $u$ given $Education$.
- The model
$$Wages = \beta_0 + \beta_1 Education + u$$
might be misspecified.
- Confounding Effects (omitted factors): for instance
$$Wages = \beta_0 + \beta_1 Education + \beta_2 Experience + u$$
- Endogeneity.

David Hendry's *3 Golden rules of Econometrics*:

1. Test.
2. Test.
3. Test.

- Cross Sectional.
- Time Series.
- Panel

- Cross-sectional data is usually a random sample.
- Each observation is a new individual, household, firm, etc.. with information at a point in time.
- **Examples:** Data on expenditures, income, hours of work, household composition, assets, investments, employment, etc..
- If the data is not a random sample, we have a sample-selection problem.

**A Cross-Sectional Data Set on Wages and Other Individual Characteristics**

| obsno | wage | educ | exper | female | married |
|---|---|---|---|---|---|
| 1 | 3.10 | 11 | 2 | 1 | 0 |
| 2 | 3.24 | 12 | 22 | 1 | 1 |
| 3 | 3.00 | 11 | 2 | 0 | 0 |
| 4 | 6.00 | 8 | 44 | 0 | 1 |
| 5 | 5.30 | 12 | 7 | 0 | 1 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 525 | 11.56 | 16 | 5 | 0 | 1 |
| 526 | 3.50 | 14 | 5 | 1 | 0 |

- Time series data has a separate observation for each time period.
- Typically Macroeconomic measures: GDP, Inflation, Prices, Exchange Rates, Interest Rates, etc..
- Financial data: Stock Prices, Bonds and other financial instruments at frequencies that range from minute to minute up to annual (useful to analyse financial markets).
- Since not a random sample, different problems to consider.
- Trends and seasonality will be important.

**Minimum Wage, Unemployment, and Related Data for Puerto Rico**

| obsno | year | avgmin | avgcov | unemp | gnp |
|---|---|---|---|---|---|
| 1 | 1950 | 0.20 | 20.1 | 15.4 | 878.7 |
| 2 | 1951 | 0.21 | 20.7 | 16.0 | 925.0 |
| 3 | 1952 | 0.23 | 22.6 | 14.8 | 1015.9 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 37 | 1986 | 3.35 | 58.1 | 18.9 | 4281.6 |
| 38 | 1987 | 3.35 | 58.2 | 16.8 | 4496.7 |

- Can follow the same random individual observations over time – known as panel data or longitudinal data.
- Used to study dynamic aspects of household and firm behaviour and to measure the impact of variables that vary predominantly over time.

**A Two-Year Panel Data Set on City Crime Statistics**

| obsno | city | year | murders | population | unem | police |
|---|---|---|---|---|---|---|
| 1 | 1 | 1986 | 5 | 350000 | 8.7 | 440 |
| 2 | 1 | 1990 | 8 | 359200 | 7.2 | 471 |
| 3 | 2 | 1986 | 2 | 64300 | 5.4 | 75 |
| 4 | 2 | 1990 | 1 | 65100 | 5.5 | 75 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 297 | 149 | 1986 | 10 | 260700 | 9.6 | 286 |
| 298 | 149 | 1990 | 6 | 245000 | 9.8 | 334 |
| 299 | 150 | 1986 | 25 | 543000 | 4.3 | 520 |
| 300 | 150 | 1990 | 32 | 546200 | 5.2 | 493 |

$$E\left[y|x\right] = \beta_0 + \beta_1 x$$

or equivalently

$$\begin{aligned} y &= \beta_0 + \beta_1 x + u, \\ E\left[u|x\right] &= 0. \end{aligned}$$

In the model:

- $\beta_0$ is known as the *intercept parameter* or *constant term*.
- $\beta_1$ is known as the *slope parameter*.

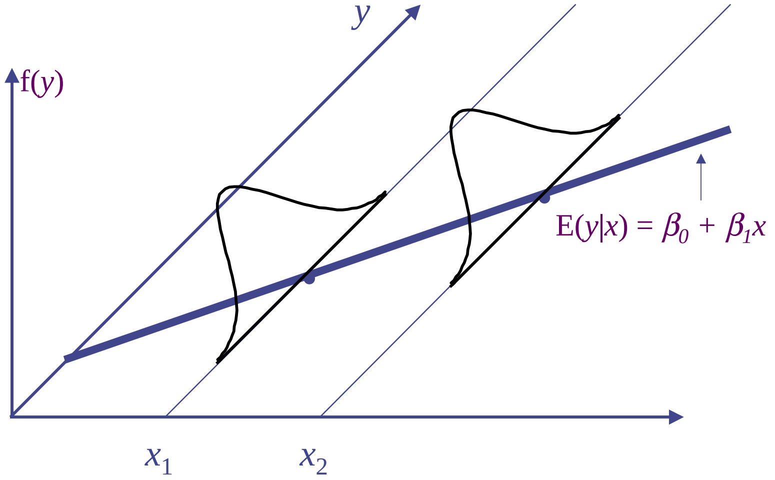
**Terminology for Simple Regression**

| $y$ | $x$ |
|---|---|
| Dependent variable | Independent variable |
| Explained variable | Explanatory variable |
| Response variable | Control variable |
| Predicted variable | Predictor variable |
| Regressand | Regressor |

$$\begin{aligned} y &= \beta_0 + \beta_1 x + u, \\ E\left[u|x\right] &= 0. \end{aligned}$$

- $\beta_0 + \beta_1 x$ is the *systematic part* of $y$.
- $u$, the error term, is the *unsystematic part* of $y$.

$E(y|x)$ as a linear function of $x$, where for any $x$ the distribution of $y$ is centered about $E(y|x)$.



Basic idea of regression is to estimate the population parameters from a sample.

- Let $\{(x_i, y_i) : i = 1, \ldots, n\}$ denote a random sample of size $n$ from the population.
- For each observation in this sample, it will be the case that

$$y_i = \beta_0 + \beta_1 x_i + u_i.$$

- To derive the OLS estimates we need to realize that our main assumption of $E(u|x) = 0$ also implies that

$$\begin{aligned} E(u) &= 0, \\ Cov(x,u) &= E(xu) = 0. \end{aligned}$$

- Why? Because of the *Law of Iterated Expectations*.
- **Proof:**

$$\begin{aligned} E(u) &= E[E(u|x)] \\ &= E(0) \\ &= 0. \end{aligned}$$

- **Proof (cont):**

$$\begin{aligned}
Cov(x,u) &= E(xu) - E(x)E(u) \\
&= E(xu) - E(x) \times 0 \\
&= E(xu) \\
&= E[E(xu|x)] \\
&= E[xE(u|x)] \\
&= E[x \times 0] \\
&= E(0) \\
&= 0.
\end{aligned}$$

We can write our 2 restrictions just in terms of $x$, $y$, $\beta_0$ and $\beta_1$ , since $u = y - \beta_0 - \beta_1 x$ :

$$\begin{aligned} E(y - \beta_0 - \beta_1 x) &= 0, \\ E[x(y - \beta_0 - \beta_1 x)] &= 0. \end{aligned}$$

These are called *moment restrictions*.

- We use the *Method of moments* to propose an estimator for the parameters $\beta_0$ and $\beta_1$. The moment restrictions

$$\begin{aligned} E(y-\beta_0-\beta_1 x) &= 0, \\ E[x(y-\beta_0-\beta_1 x)] &= 0, \end{aligned}$$

  correspond to population means of random variables. Hence the estimator suggested by the Method of Moments is obtained if we replace population means by sample means.
- What does this mean? Recall that for $E(X)$, the mean of a population distribution, a sample estimator of $E(X)$ is simply the arithmetic mean of the sample $\bar{X} = \sum_{i=1}^{n} X_i/n$.

- The moment restrictions in the population:

$$
\begin{aligned}
E(y - \beta_0 - \beta_1 x) &= 0, \\
E[x(y - \beta_0 - \beta_1 x)] &= 0.
\end{aligned}
$$

- We want to choose values of the parameters that will ensure that the sample versions of our moment restrictions are true. The sample versions are as follows:

$$
\frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) = 0, \quad (1)
$$

$$
\frac{1}{n} \sum_{i=1}^{n} x_i (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) = 0. \quad (2)
$$

The OLS estimator is given by the pair $(\hat{\beta}_0, \hat{\beta}_1)$ that solves these equations.

Solving equation $(1)$ we have

$$\begin{aligned} 0 &= \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i\right) \\ &= \frac{1}{n}\sum_{i=1}^{n} y_i - \frac{1}{n}\sum_{i=1}^{n}\hat{\beta}_0 - \frac{1}{n}\sum_{i=1}^{n}\hat{\beta}_1 x_i, \\ &= \bar{y} - \frac{1}{n}\left(\underbrace{\hat{\beta}_0 + \hat{\beta}_0 + \ldots + \hat{\beta}_0}_{n\times}\right) - \hat{\beta}_1 \frac{1}{n}\sum_{i=1}^{n} x_i \\ &= \bar{y} - \frac{n}{n}\hat{\beta}_0 - \hat{\beta}_1 \bar{x} \\ &= \bar{y} - \hat{\beta}_0 - \hat{\beta}_1 \bar{x}, \end{aligned}$$

where $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$,
Therefore

$$\bar{y} - \hat{\beta}_0 - \hat{\beta}_1 \bar{x} = 0,$$
$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} \qquad (3)$$

Plugging (3) into (2) we have

$$\begin{aligned} 0 &= \frac{1}{n}\sum_{i=1}^{n} x_i(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) \\ &= \sum_{i=1}^{n}\left[y_i - (\bar{y} - \hat{\beta}_1\bar{x}) - \hat{\beta}_1 x_i\right] x_i \\ &= \sum_{i=1}^{n}\left[y_i - \bar{y} + \hat{\beta}_1\bar{x} - \hat{\beta}_1 x_i\right] x_i \\ &= \sum_{i=1}^{n}\left[(y_i - \bar{y}) + \hat{\beta}_1(\bar{x} - x_i)\right] x_i \\ &= \sum_{i=1}^{n}\left[(y_i - \bar{y}) x_i + \hat{\beta}_1(\bar{x} - x_i) x_i\right] \\ &= \sum_{i=1}^{n} x_i(y_i - \bar{y}) - \sum_{i=1}^{n}\hat{\beta}_1 x_i(x_i - \bar{x}) \\ &= \sum_{i=1}^{n} x_i(y_i - \bar{y}) - \hat{\beta}_1\sum_{i=1}^{n} x_i(x_i - \bar{x}) \end{aligned}$$

which leads to $\hat{\beta}_2 = \dfrac{\frac{1}{n}\sum_{i=1}^{n} x_i(y_i - \bar{y})}{\frac{1}{n}\sum_{i=1}^{n} x_i(x_i - \bar{x})} = \dfrac{\sum_{i=1}^{n} x_i(y_i - \bar{y})}{\sum_{i=1}^{n} x_i(x_i - \bar{x})}$.

We prove now that

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} x_i (y_i - \bar{y})}{\sum_{i=1}^{n} x_i (x_i - \bar{x})} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

$$\begin{aligned}
\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) &= \sum_{i=1}^{n} [x_i (y_i - \bar{y}) - \bar{x}(y_i - \bar{y})] \\
&= \sum_{i=1}^{n} x_i (y_i - \bar{y}) - \sum_{i=1}^{n} \bar{x}(y_i - \bar{y})
\end{aligned}$$

$$\begin{aligned}
\sum_{i=1}^{n} \bar{x}(y_i - \bar{y}) &= \bar{x} \sum_{i=1}^{n} (y_i - \bar{y}) \\
&= \bar{x} \left[ \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} \bar{y} \right] \\
&= \bar{x} \left[ n \frac{1}{n} \sum_{i=1}^{n} y_i - \left( \underbrace{\bar{y} + \bar{y} + \ldots + \bar{y}}_{n\times} \right) \right] \\
&= \bar{x} [n\bar{y} - n\bar{y}] \\
&= 0
\end{aligned}$$

$$
\begin{aligned}
\sum_{i=1}^{n}(x_i-\bar{x})^2 &= \sum_{i=1}^{n}(x_i-\bar{x})(x_i-\bar{x}) \\
&= \sum_{i=1}^{n}\left[x_i(x_i-\bar{x})-\bar{x}(x_i-\bar{x})\right] \\
&= \sum_{i=1}^{n}x_i(x_i-\bar{x})-\sum_{i=1}^{n}\bar{x}(x_i-\bar{x})
\end{aligned}
$$

$$
\begin{aligned}
\sum_{i=1}^{n}\bar{x}(x_i-\bar{x}) &= \bar{x}\sum_{i=1}^{n}(x_i-\bar{x}) \\
&= \bar{x}\left[\sum_{i=1}^{n}x_i-\sum_{i=1}^{n}\bar{x}\right] \\
&= \bar{x}\left[n\frac{1}{n}\sum_{i=1}^{n}y_i-\left(\underbrace{\bar{x}+\bar{x}+\ldots+\bar{x}}_{n\times}\right)\right] \\
&= \bar{x}\left[n\bar{x}-n\bar{x}\right] \\
&= 0.
\end{aligned}
$$

The solution of this system of equations is given by

$$\begin{aligned}\hat{\beta}_0 &= \bar{y} - \hat{\beta}_1 \bar{x}, \\ \hat{\beta}_1 &= \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2},\end{aligned}$$

where $\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$, $\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$, and it is assumed that $\sum_{i=1}^{n} (x_i - \bar{x})^2 > 0$.
The residual is defined as $\hat{u}_i = y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i$.

- There is an alternative justification for this estimator that justifies its name.
- This estimator is known as *Ordinary Least Squares* estimator because it is fitting a line through the sample points such that the mean of squared residuals is as small as possible.
- Consider the function

$$S = \frac{1}{n} \sum_{i=1}^{n} \left(y_i - b_0 - b_1 x_i\right)^2 .$$

- This function takes its minimum when $b_0 = \hat{\beta}_0$ and $b_1 = \hat{\beta}_1$.
- To see this notice that by using calculus to solve the minimization problem for the two parameters you obtain the following first order conditions:

$$\begin{cases} \frac{\partial S}{\partial b_0} = -\frac{2}{n} \sum_{i=1}^{n} (y_i - b_0 - b_1 x_i) = 0 \\ \frac{\partial S}{\partial b_1} = -\frac{2}{n} \sum_{i=1}^{n} x_i (y_i - b_0 - b_1 x_i) = 0 \end{cases} .$$

- These conditions are the same as we obtained before, multiplied by $-2$. Hence the solution is the same: $b_0 = \hat{\beta}_0$ and $b_1 = \hat{\beta}_1$

- The the *fitted values* are defined as

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i,$$

$i = 1, ..., n.$

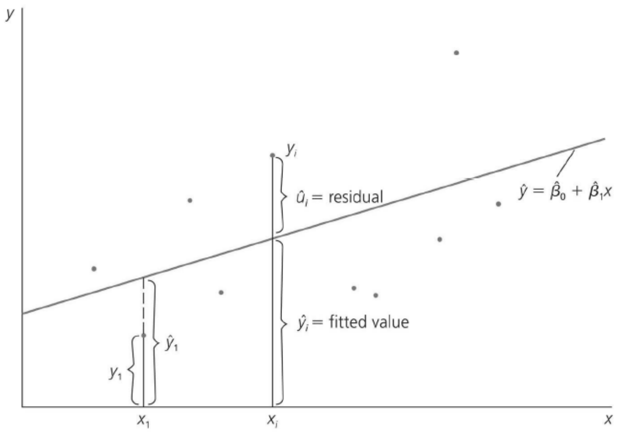
- The *residual*, $\hat{u}_i$ is the difference between the sample point and the fitted line (sample regression function)

$$\begin{aligned}\hat{u}_i &= y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i \\ &= y_i - \hat{y}_i,\end{aligned}$$

$i = 1, ..., n.$

Sample regression line (fitted values), sample data points and the associated estimated error terms:

Fitted values and residuals.



Note the differences:

- Population regression line

$$E\left[y_i|x_i\right] = \beta_0 + \beta_1 x_i$$

  $i = 1, ..., n.$

- Sample regression line (fitted values)

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i,$$

  $i = 1, ..., n.$

**A Note on Terminology:** Often we indicate that the equation

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i,$$

$i = 1, ..., n$, was obtained by OLS by saying that we run a regression of $y$ on $x$, or that we regress $y$ on $x$.

**Example:**

- Regression of Wages on Education

Dependent valiable: Wages

Estimation Method: Ordinary Least Squares

Sample size: 528

| Regressors | Estimates |
|---|---|
| Intercept | $-1.60468$ |
| Education | 0.81395 |

Hence the fitted values are equal to

$$\widehat{Wages} = -1.60468 + 0.81395 \times Education.$$

**Interpretation:**

- This means that one extra year of schooling increases the average hourly wages by \$0.81395.
- The results should be interpreted with caution as the intercept of $-1.60468$ means that the average hourly wages of people with no education is $-1.60468$ which does not make sense. In the sample we do not have people with less than 6 years of education and in this case $\widehat{Wages} = -1.60468 + 0.81395 \times 6 = 3.279$.

## Scatterplot and the sample regression line - Hourly wages (in dollars)